

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER B.TECH DEGREE EXAMINATION(S), DECEMBER 2019

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks.

Marks

- 1 a) Find the sum of the series $\sum_{k=1}^{\infty} \frac{2}{3^{(k+1)}}$ (2)
- b) Determine whether the alternating series $\sum_{k=2}^{\infty} (-1)^k \frac{k}{k-1}$ converges. (3)
- 2 a) Find the slope of the function $f(x, y) = x \cos(xy) + y \sin(xy)$ at $(\pi, 1)$ along the x- direction. (2)
- b) If $z = f(x^2 - y^2)$, show that

$$y \frac{\partial z}{\partial x} + x \frac{\partial z}{\partial y} = 0$$
 (3)
- 3 a) Find $\lim_{t \rightarrow 0} \mathbf{r}(t)$, where $\mathbf{r}(t) = \langle 1 + t^3, te^{-t}, \frac{\sin t}{t} \rangle$ (2)
- b) Find the directional derivative of $f(x, y) = e^x \cos y$ at $P(0, \pi/4)$ in the direction of negative Y-axis (3)
- 4 a) Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z} dx dy dz$ (2)
- b) Evaluate $\iint_R (x^2 + y^2) dx dy$ where R is the region taken over the first quadrant for which $x + y \leq 1$. (3)
- 5 a) Find the divergence of the vector field
 $\mathbf{F}(x, y, z) = x^2y \mathbf{i} + 2y^3z \mathbf{j} + 3z \mathbf{k}$ (2)
- b) Evaluate $\int_C x^2 dy + y^2 dx$ where C is the path $y = x$ from (0,0) to (1,1) (3)
- 6 a) Determine the source and sink of the vector field
 $\mathbf{F}(x, y, z) = 2(x^3 - 2x)\mathbf{i} + 2(y^3 - 2y)\mathbf{j} + 2(z^3 - 2z)\mathbf{k}$ (2)
- b) If S is any closed surface enclosing a volume V and if
 $\mathbf{A} = ax\mathbf{i} + by\mathbf{j} + cz\mathbf{k}$ prove that $\iiint_V \nabla \cdot \mathbf{A} \, dV = \iint_S \mathbf{A} \cdot \mathbf{n} \, dS = (a + b + c) V$ (3)

PART B**Module 1****Answer any two questions, each carries 5 marks.**

- 7 Test for convergence of the series $\sum_{k=0}^{\infty} \frac{(k!)^2}{(2k)!}$. (5)
- 8 Find the radius of convergence of $\sum_{k=0}^{\infty} \frac{(2k-1)^k}{3^{2k}}$. (5)
- 9 Expand $f(x) = \sin \pi x$ into a Taylors series about $x = \frac{1}{2}$, up to third derivative. (5)

Module 1I**Answer any two questions, each carries 5 marks.**

- 10 If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$. (5)
- 11 Find the local linear approximation $L(x, y)$ of $f(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$ at the point $P(4, 3)$. Compare the error in the approximation to f by L at the point $Q(3.92, 3.01)$ with the distance between P and Q . (5)
- 12 Locate all relative extrema and saddle point for the function $f(x, y) = x^3 + y^3 - 6xy + 20$. (5)

Module 1II**Answer any two questions, each carries 5 marks.**

- 13 Find the equation of the unit tangent and unit normal to the curve $x = e^t \cos t, y = e^t \sin t, z = e^t$; at $t = 0$. (5)
- 14 A particle moves along the curve $r(t) = \left(\frac{1}{t}\right)i + t^2j + t^3k$, where t denotes time. Find
 1) The scalar tangential and normal components of acceleration at time $t = 1$. (5)
 2) The vector tangential and normal component of acceleration at time $t = 1$
- 15 Find the equation of the tangent plane and the parametric equations of the normal line to the surface $z = 4x^3y^2 + 2y - 2$ at $(1, -2, 10)$. (5)

Module 1V**Answer any two questions, each carries 5 marks.**

- 16 Use double integral to find the area of the plane enclosed by $y^2 = 4x$ and $x^2 = 4y$ (5)
- 17 Change the order of integration to evaluate $\int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$ (5)

- 18 Use triple integral to find the volume of the solid with in the cylinder $x^2 + y^2 = 4$ and between the planes $z = 0$ and $y + z = 3$. (5)

Module V

Answer any three questions, each carries 5 marks.

- 19 If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\vec{r}|$, prove that $\nabla^2 r^n = n(n+1)r^{n-2}$ (5)
- 20 Evaluate $\int_C (3x^2 + y^2) dx + 2xydy$ along the curve
 $C: x = \cos t, y = \sin t, 0 \leq t \leq \frac{\pi}{2}$ (5)
- 21 Find the scalar potential of $\vec{F} = (2xy + z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$ (5)
- 22 Find the work done by $F(x, y) = (x + y)\vec{i} + xy\vec{j} - z^2\vec{k}$ along the line segments from $(0, 0, 0)$ to $(1, 3, 1)$ to $(2, -1, 5)$ (5)
- 23 Show that $\int_{(0,0)}^{(1,\frac{\pi}{2})} e^x \sin y dx + e^x \cos y dy$ is independent of path. (5)
 Hence evaluate $\int_{(0,0)}^{(1,\frac{\pi}{2})} e^x \sin y dx + e^x \cos y dy$

Module VI

Answer any three questions, each carries 5 marks.

- 24 Evaluate using Green's theorem in the plane $\int_C (x^2 dx - xydy)$ where C is the boundary of the square formed by $x = 0, y = 0, x = a, y = a$ (5)
- 25 Evaluate the surface integral $\iint_{\sigma} f(x, y, z) ds$ where $f(x, y, z) = x + y$, σ is the portion of the surface $z = 6 - 2x - 4y$ in the first octant. (5)
- 26 Using divergence theorem find the flux across the surface σ which is the surface of the tetrahedron in the first octant bounded by $x + y + z = 1$ and the coordinate planes, $\vec{F} = (x^2 + y)\vec{i} + xy\vec{j} - (2xz + y)\vec{k}$ (5)
- 27 Evaluate $\int_C (e^x dx + 2ydy - dz)$ where C is the curve $x^2 + y^2 = 4, z = 2$ using Stoke's theorem (5)
- 28 Evaluate the surface integral $\iint_{\sigma} f(x, y, z) ds$ where $f(x, y, z) = x^2 + y^2$, σ is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ (5)
